Math 3150 Exam 3A B, April 2026, 13 problems, 75 minutes

Name: ANSWERS

Read these instructions:

- Leaving the testing room results in a new exam given for the unfinished problems.
- Two detached sheets of notes allowed.
- No electronics.
- Raise your hand for questions or more paper.

Problem 1. Write the following statements in terms of the letters $a =$ "Ann is a math major", $b =$ "Ben is a math major", $d =$ "Dan is a math major" and the symbols $\neg$, $\wedge$, $\vee$, $\oplus$.

Part A. "Either Dan or Ben is a math major." $d \vee b$ OR $d \oplus b$.

Part B. "Neither Dan nor Ann are math majors." $\neg(d \vee a) \equiv \neg d \wedge \neg a$

Part C. "Ben is not a math major." $\neg b$

Problem 2A. State the **converse** of "If there is smoke, then there is fire" in English.

If there is fire, then there is smoke

Problem 2B. State the **contrapositive** of "If there is smoke, then there is fire" in English.

If there is no fire, then there is no smoke

Problem 3. Find a disjunctive normal form for $(p \Rightarrow q) \Rightarrow q$.

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F

so a DNF for $(p \Rightarrow q) \Rightarrow q$ is $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$.

Other answer: $p \vee q$

Problem 4. Is $(p \Rightarrow q) \wedge \neg p$ logically equivalent to $\neg p$? Explain.

p	q	$p \Rightarrow q$	$\neg p$	$(p \Rightarrow q) \wedge (\neg p)$
T	T	T	F	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Final two columns are equal, so they are logically equivalent.

Problem 5. Let $T(x) =$"x has thorns", $P(x) =$"x has petals", and R be the set of all roses. Write "Every rose has thorns but some rose have petals" in terms of x, $T(x)$, $P(x)$, R, $\forall$, $\exists$, $\in$, :, [,], $\wedge$, $\vee$, $\neg$, $\Rightarrow$.

$$[\forall x \in R : T(x)] \wedge [\exists x \in R : P(x)]$$

Problem 6. Let $B(x) =$ "x has at least one bird", $C(x) =$ "x has at least one cat", $D(x) =$ "x has at least one dog", and S be the set of students in this class. Formalize (a)-(c) via the symbols
$S, x, B(x), C(x), D(x), \wedge, \vee, \neg, \Rightarrow, \forall, \exists, :$ (colon), $\in, [,]$.

+2 **Part A.** Every student in this class has at least one bird or at least one cat.

$\forall x \in S : B(x) \vee C(x)$

+2 **Part B.** Some student in this class has at least one bird, at least one cat, or at least one dog.

$\exists x \in S : B(x) \vee C(x) \vee D(x)$

+2 **Part C.** Every student in this class has at most one type of the animals out of birds, cats, and dogs.

$\forall x \in S : [\neg B(x) \wedge \neg C(x)] \vee [\neg B(x) \wedge \neg D(x)] \vee [\neg C(x) \wedge \neg D(x)]$.

+2 **Problem 7.** Which option below is equivalent to the **negation** of "Both Abe and Ben are tall"?

(a) Abe is tall, and Ben is tall.
(b) Abe is not tall, and Ben is not tall.
(c) Abe is tall, or Ben is tall.
(d) Abe is not tall, or Ben is not tall.

~~either~~ or not tall

Problem 8. Mark each item as true or false. Justify your answers:

+2 **Part A.** $\forall x \in \mathbb{Z} : x^2 \geq x$ True : $0^2 \geq 0$ ✓, $1^2 \geq 1$ ✓, $2^2 \geq 2$ ✓, $(-1)^2 \geq -1$ ✓, $(-2)^2 \geq -2$ ✓

+2 **Part B.** $\exists x \in \mathbb{Q} : x^2 < x$ True : let $x = \frac{1}{2}$: $\left(\frac{1}{2}\right)^2 = \frac{1}{4} < \frac{1}{2}$

+2 **Part C.** $\forall x \in \mathbb{R} : [\exists y \in \mathbb{R} : x + y^2 = 0]$ False : for $x = +1$, $1 + y^2 = 0 \Rightarrow y^2 = -1$ has no real solution.

+2 **Part D.** $\forall y \in \mathbb{R} : [\exists x \in \mathbb{R} : xy > 0]$ False : let $y = 0$, then $xy = x \cdot 0 = 0 \not> 0$ no matter the x.

+2 **Part E.** $\exists x \in \mathbb{R} : [(x > 0) \Rightarrow (x^2 = -1)]$ True : let $x = 0$ then

$[(x > 0) \Rightarrow (x^2 = -1)]$
$\equiv [\ F \Rightarrow F\ ]$
$\equiv T$.

+2 **Problem 9.** Is the function $f : \{a, b, c, d\} \to \{u, v, w, x, y\}$ defined by $f(a) = y$, $f(b) = x$, $f(c) = w$, $f(d) = v$ onto? Explain.

No: u is not an output.

+4 **Problem 10A.** Is the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2(x + 1)$ one-to-one? Explain.

Yes: let $y \in \text{codomain}(g) = \mathbb{R}$. Then $y = g(x) \Rightarrow y = 2(x+1) \overset{\div 2}{\Rightarrow} \frac{y}{2} = x + 1 \overset{-1}{\Rightarrow} x = \frac{y}{2} - 1 \in \mathbb{R} = \text{domain}(g)$

+4 **Problem 10B.** Is the function $h : \mathbb{Z} \to \mathbb{Z}$ defined by $h(x) = 2(x + 1)$ onto? Explain.

No: 1 is not an output since $1 = h(x) \Rightarrow 1 = 2(x+1) \overset{\div 2}{\Rightarrow} \frac{1}{2} = x + 1 \overset{-1}{\Rightarrow} x = -\frac{1}{2} \notin \mathbb{Z} = \text{domain}(h)$.

Problem 11. Suppose that p and q are statements such that $p \Rightarrow q$ is true. ← occurs when $p = T, q = T$ or $p = F, q = T$ or $p = F, q = F$.

+2 **Part A.** Find the truth value of $p \vee q$. ← T or F

+2 **Part B.** Find the truth value of $p \wedge \neg q$. ← F

p	q	$p \Rightarrow q$	$p \vee q$	$p \wedge \neg q$	$\neg q$
T	T	T	T	F	F
~~T~~	~~F~~	~~F~~	~~T~~	~~T~~	~~T~~
F	T	T	T	F	F
F	F	T	F	F	T

Problem 12. Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n^2$.

+3 **Part A.** Is the statement $\underbrace{\forall x \in \mathbb{N} : [\forall y \in \mathbb{N} : (f(x) = f(y)) \Rightarrow (x = y)]}_{\text{"}f : \mathbb{N} \to \mathbb{N} \text{ is one-to-one"}}$ true or false? Explain.

True: f is one-to-one as a function $\mathbb{N} \to \mathbb{N}$ since $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \overset{\sqrt{\cdot}}{\Rightarrow} x_1 = \pm x_2 \underset{x_1, x_2 \geq 0}{\Rightarrow} x_1 = x_2 \Rightarrow f$ is one-to-one.

+3 **Part B.** Is the statement $\underbrace{\forall y \in \mathbb{N} : [\exists x \in \mathbb{N} : y = f(x)]}_{\text{"}f : \mathbb{N} \to \mathbb{N} \text{ is onto"}}$ true or false? Explain.

False: $f : \mathbb{N} \to \mathbb{N}$ is not onto: 2 is not an output since $2 = f(x) = x^2 \overset{\sqrt{\cdot}}{\Rightarrow} x = \sqrt{2} \notin \mathbb{N}$.

Page	1	2	3	4
Score	22	18		

40